

```

SetDirectory["C:/drorbn/AcademicPensieve/Projects/Arrow_Diagrams_and_gl(N)"];
<< "Arrow_Diagrams_and_gl(N).m"
base = 10;

diag = BasisAArrow[4][[120]]
Diag[ar[4, 7], ar[5, 3], ar[6, 2], ar[8, 1]]

diag3 = BasisAArrow[3][[12]]
Diag[ar[3, 4], ar[5, 1], ar[6, 2]]

UGLnBiAlg[diag3]

```

A very large output was generated. Here is a sample of it:

$$\begin{aligned}
& -\frac{1}{8} n h[1]^2 RW[] + 4 n^2 RW[11] + 10 n^3 RW[11] + \ll 2397 \gg + 6 n^5 RW[15, 25, 34, 43, 51, 52] + \\
& 6 n^6 RW[15, 26, 34, 43, 51, 62] + 6 n^6 RW[16, 23, 32, 45, 54, 61] + \\
& 6 n^6 RW[16, 24, 35, 42, 53, 61] + 6 n^6 RW[16, 25, 34, 43, 52, 61]
\end{aligned}$$

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```

U[deg_, diag_] := Expand[Plus[
  UGLnBiAlg[diag] /. n^p_.*RW[l_] => EW[n^p*FromDigits[{1}, base^2]] //.
  h[i_]^p_.EW[l_] => EW[h[i]^p l],
  Coproduct[diag, 1] /. T[d1_, d2_] => Distribute[T[UGLnBiAlg[d1], UGLnBiAlg[d2]]] /.
  T[c_?NumberQ * a1_, a2_] => c*T[a1, a2] /.
  T[a1_, c_?NumberQ * a2_] => c*T[a1, a2] /. T[a1_, a2_] => EW[1, a1, a2],
  Coproduct[diag, 2] /. T[d1_, d2_] => Distribute[T[UGLnBiAlg[d1], UGLnBiAlg[d2]]] /.
  T[c_?NumberQ * a1_, a2_] => c*T[a1, a2] /.
  T[a1_, c_?NumberQ * a2_] => c*T[a1, a2] /. T[a1_, a2_] => EW[2, a1, a2]
]]
U[3, diag3]

```

A very large output was generated. Here is a sample of it:

$$\begin{aligned}
& -\frac{EW[0]}{8} + \frac{1}{8} EW[111111111111n] + 4 EW[11n^2] - \\
& 4 EW[22n^2] + \ll 3561 \gg + 4 EW[2, n^4 RW[14, 23, 32, 41], n^2 RW[22]] + \\
& 3 EW[2, n^4 RW[14, 23, 32, 41], n RW[11, 11]] + 6 EW[2, n^4 RW[14, 23, 32, 41], n^2 RW[12, 21]]
\end{aligned}$$

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```
vs = U[3, #] & /@ Diagrams[3 ar]
```

A very large output was generated. Here is a sample of it:

$$\left\{ -\frac{EW[0]}{8} + \frac{1}{8} EW[111\ 111\ 111\ 111\ n] + 4 EW[1221\ n^2] - \right.$$

$$4 EW[111\ 221\ n^2] + \ll 855 \gg + 6 EW[2, n^4 RW[13, 24, 31, 42], n^2 RW[12, 21]] -$$

$$3 EW[2, n^4 RW[14, 23, 32, 41], n h[1]^2 RW[]] + 3 EW[2, n^4 RW[14, 23, 32, 41], n RW[11, 11]] +$$

$$6 EW[2, n^4 RW[14, 23, 32, 41], n^2 RW[12, 21]], \ll 118 \gg, -\frac{\ll 1 \gg}{8} + \ll 3776 \gg + 6 \ll 1 \gg \left. \right\}$$
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```
EWs = Union[Cases[vs, _EW, Infinity]]
```

A very large output was generated. Here is a sample of it:

$$\{ EW[0], EW[111\ 111\ 111\ 111\ n], EW[11\ n^2], EW[22\ n^2],$$

$$EW[1111\ n^2], \ll 2844 \gg, EW[2, n^4 RW[14, 23, 32, 41], n h[1]^2 RW[]],$$

$$EW[2, n^4 RW[14, 23, 32, 41], n^2 RW[11]], EW[2, n^4 RW[14, 23, 32, 41], n^2 RW[22]],$$

$$EW[2, n^4 RW[14, 23, 32, 41], n RW[11, 11]], EW[2, n^4 RW[14, 23, 32, 41], n^2 RW[12, 21]] \}$$
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```
EWRule = Dispatch[Thread[EWs → Array[v, {Length[EWs]}]]]
```

A very large output was generated. Here is a sample of it:

$$\text{Dispatch}[\{ EW[0] \rightarrow v[1], EW[111\ 111\ 111\ 111\ n] \rightarrow v[2],$$

$$EW[11\ n^2] \rightarrow v[3], \ll 2848 \gg, EW[2, n^4 RW[14, 23, 32, 41], n^2 RW[22]] \rightarrow v[2852],$$

$$EW[2, n^4 RW[14, 23, 32, 41], n RW[11, 11]] \rightarrow v[2853],$$

$$EW[2, n^4 RW[14, 23, 32, 41], n^2 RW[12, 21]] \rightarrow v[2854] \}, -\text{DispatchTables} -]$$
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```
Total[Head /@ vs]
```

```
120 Plus
```

```
mat = SparseArray[
  Join @@ Table[
    t = List @@ (vs[[i]] /. EWRule);
    Replace[#, c_.*v[j_] => ({i, j} → c)] & /@ t,
    {i, Length[vs]}
  ],
  {Length[vs], Length[EWs]}
]
```

```
SparseArray[<248 814>, {120, 2854}]
```

```
MatrixRank[mat]
```

```
27
```

```
ds = Take[BasisAArrow[4], 2];
vs = Table[
  Print[i, " ", ds[[i]]];
  U[4, ds[[i]],
  {i, Length[ds]}
];
Print["Finding EWs..."];
EWS = Union[Cases[vs, _EW, Infinity]];
EWRule = Dispatch[Thread[EWS → Array[v, {Length[EWS]}]]];
Print["Computing mat..."];
Print[{Length[vs], Length[EWS]}];
mat = SparseArray[
  Join @@ Table[
    t = List @@ (vs[[i]] /. EWRule);
    Replace[#, c_. * v[j_] => ({i, j} → c)] & /@ t,
    {i, Length[vs]}
  ],
  {Length[vs], Length[EWS]}
];
Print["Computing Rank..."];
MatrixRank[mat]
```

```
1 Diag[ar[5, 1], ar[6, 4], ar[7, 3], ar[8, 2]]
```

```
2 Diag[ar[3, 2], ar[6, 1], ar[7, 5], ar[8, 4]]
```

```
Finding EWs...
```

```
Computing mat...
```

```
{2, 63114}
```

```
Computing Rank...
```

```
2
```

```

ds = Take[BasisAArrow[4], All];
vs = Table[
  Print[i, " ", ds[[i]]];
  U[4, ds[[i]]],
  {i, Length[ds]}
];
Print["Finding EWs..."];
EWS = Union[Cases[vs, _EW, Infinity]];
EWRule = Dispatch[Thread[EWS → Array[v, {Length[EWS]}]]];
Print["Computing mat..."];
Print[{Length[vs], Length[EWS]}];
mat = SparseArray[
  Join @@ Table[
    t = List @@ (vs[[i]] /. EWRule);
    Replace[#, c_. * v[j_] => ({i, j} → c)] & /@ t,
    {i, Length[vs]}
  ],
  {Length[vs], Length[EWS]}
];
Print["Computing Rank..."];
MatrixRank[mat]
1 Diag[ar[5, 1], ar[6, 4], ar[7, 3], ar[8, 2]]
2 Diag[ar[3, 2], ar[6, 1], ar[7, 5], ar[8, 4]]
3 Diag[ar[4, 2], ar[6, 1], ar[7, 5], ar[8, 3]]
4 Diag[ar[4, 7], ar[5, 2], ar[6, 1], ar[8, 3]]
5 Diag[ar[5, 2], ar[6, 1], ar[7, 4], ar[8, 3]]
6 Diag[ar[4, 3], ar[6, 1], ar[7, 5], ar[8, 2]]
7 Diag[ar[4, 7], ar[5, 3], ar[6, 1], ar[8, 2]]
8 Diag[ar[5, 3], ar[6, 1], ar[7, 4], ar[8, 2]]
9 Diag[ar[4, 5], ar[6, 1], ar[7, 3], ar[8, 2]]
10 Diag[ar[5, 4], ar[6, 1], ar[7, 3], ar[8, 2]]
11 Diag[ar[1, 7], ar[4, 2], ar[6, 3], ar[8, 5]]
12 Diag[ar[1, 7], ar[3, 6], ar[4, 8], ar[5, 2]]
13 Diag[ar[1, 7], ar[3, 6], ar[5, 2], ar[8, 4]]
14 Diag[ar[1, 7], ar[5, 2], ar[6, 3], ar[8, 4]]
15 Diag[ar[1, 7], ar[3, 8], ar[5, 2], ar[6, 4]]
16 Diag[ar[1, 7], ar[5, 2], ar[6, 4], ar[8, 3]]
17 Diag[ar[1, 7], ar[2, 6], ar[5, 3], ar[8, 4]]
18 Diag[ar[1, 7], ar[4, 3], ar[6, 2], ar[8, 5]]
19 Diag[ar[1, 7], ar[3, 5], ar[4, 8], ar[6, 2]]
20 Diag[ar[1, 7], ar[3, 5], ar[6, 2], ar[8, 4]]
21 Diag[ar[1, 7], ar[4, 8], ar[5, 3], ar[6, 2]]

```

```
22 Diag[ar[1, 7], ar[5, 3], ar[6, 2], ar[8, 4]]
23 Diag[ar[1, 7], ar[3, 8], ar[4, 5], ar[6, 2]]
24 Diag[ar[1, 7], ar[3, 8], ar[5, 4], ar[6, 2]]
25 Diag[ar[1, 7], ar[4, 5], ar[6, 2], ar[8, 3]]
26 Diag[ar[1, 7], ar[5, 4], ar[6, 2], ar[8, 3]]
27 Diag[ar[1, 7], ar[2, 8], ar[5, 3], ar[6, 4]]
28 Diag[ar[1, 7], ar[2, 8], ar[5, 4], ar[6, 3]]
29 Diag[ar[1, 7], ar[3, 5], ar[6, 4], ar[8, 2]]
30 Diag[ar[1, 7], ar[4, 6], ar[5, 3], ar[8, 2]]
31 Diag[ar[1, 7], ar[5, 3], ar[6, 4], ar[8, 2]]
32 Diag[ar[1, 7], ar[5, 4], ar[6, 3], ar[8, 2]]
33 Diag[ar[3, 2], ar[4, 8], ar[6, 5], ar[7, 1]]
34 Diag[ar[3, 2], ar[6, 5], ar[7, 1], ar[8, 4]]
35 Diag[ar[4, 2], ar[5, 3], ar[6, 8], ar[7, 1]]
36 Diag[ar[4, 2], ar[5, 3], ar[7, 1], ar[8, 6]]
37 Diag[ar[4, 2], ar[5, 8], ar[6, 3], ar[7, 1]]
38 Diag[ar[4, 2], ar[6, 3], ar[7, 1], ar[8, 5]]
39 Diag[ar[2, 5], ar[4, 8], ar[6, 3], ar[7, 1]]
40 Diag[ar[2, 5], ar[6, 3], ar[7, 1], ar[8, 4]]
41 Diag[ar[3, 4], ar[5, 2], ar[6, 8], ar[7, 1]]
42 Diag[ar[3, 4], ar[5, 2], ar[7, 1], ar[8, 6]]
43 Diag[ar[4, 3], ar[5, 2], ar[6, 8], ar[7, 1]]
44 Diag[ar[4, 3], ar[5, 2], ar[7, 1], ar[8, 6]]
45 Diag[ar[3, 6], ar[4, 8], ar[5, 2], ar[7, 1]]
46 Diag[ar[3, 6], ar[5, 2], ar[7, 1], ar[8, 4]]
47 Diag[ar[4, 8], ar[5, 2], ar[6, 3], ar[7, 1]]
48 Diag[ar[5, 2], ar[6, 3], ar[7, 1], ar[8, 4]]
49 Diag[ar[3, 8], ar[4, 6], ar[5, 2], ar[7, 1]]
50 Diag[ar[3, 8], ar[5, 2], ar[6, 4], ar[7, 1]]
51 Diag[ar[4, 6], ar[5, 2], ar[7, 1], ar[8, 3]]
52 Diag[ar[5, 2], ar[6, 4], ar[7, 1], ar[8, 3]]
53 Diag[ar[2, 6], ar[4, 3], ar[5, 8], ar[7, 1]]
54 Diag[ar[2, 6], ar[4, 3], ar[7, 1], ar[8, 5]]
55 Diag[ar[2, 6], ar[3, 5], ar[4, 8], ar[7, 1]]
56 Diag[ar[2, 6], ar[3, 5], ar[7, 1], ar[8, 4]]
```

```
57 Diag[ar[2, 6], ar[4, 8], ar[5, 3], ar[7, 1]]
58 Diag[ar[2, 6], ar[5, 3], ar[7, 1], ar[8, 4]]
59 Diag[ar[2, 6], ar[3, 8], ar[5, 4], ar[7, 1]]
60 Diag[ar[2, 6], ar[4, 5], ar[7, 1], ar[8, 3]]
61 Diag[ar[2, 6], ar[5, 4], ar[7, 1], ar[8, 3]]
62 Diag[ar[4, 3], ar[5, 8], ar[6, 2], ar[7, 1]]
63 Diag[ar[4, 3], ar[6, 2], ar[7, 1], ar[8, 5]]
64 Diag[ar[3, 5], ar[4, 8], ar[6, 2], ar[7, 1]]
65 Diag[ar[3, 5], ar[6, 2], ar[7, 1], ar[8, 4]]
66 Diag[ar[4, 8], ar[5, 3], ar[6, 2], ar[7, 1]]
67 Diag[ar[5, 3], ar[6, 2], ar[7, 1], ar[8, 4]]
68 Diag[ar[3, 8], ar[4, 5], ar[6, 2], ar[7, 1]]
69 Diag[ar[3, 8], ar[5, 4], ar[6, 2], ar[7, 1]]
70 Diag[ar[4, 5], ar[6, 2], ar[7, 1], ar[8, 3]]
71 Diag[ar[5, 4], ar[6, 2], ar[7, 1], ar[8, 3]]
72 Diag[ar[2, 8], ar[3, 5], ar[6, 4], ar[7, 1]]
73 Diag[ar[2, 8], ar[4, 6], ar[5, 3], ar[7, 1]]
74 Diag[ar[2, 8], ar[5, 3], ar[6, 4], ar[7, 1]]
75 Diag[ar[2, 8], ar[3, 6], ar[4, 5], ar[7, 1]]
76 Diag[ar[2, 8], ar[3, 6], ar[5, 4], ar[7, 1]]
77 Diag[ar[2, 8], ar[4, 5], ar[6, 3], ar[7, 1]]
78 Diag[ar[2, 8], ar[5, 4], ar[6, 3], ar[7, 1]]
79 Diag[ar[3, 5], ar[6, 4], ar[7, 1], ar[8, 2]]
80 Diag[ar[4, 6], ar[5, 3], ar[7, 1], ar[8, 2]]
81 Diag[ar[5, 3], ar[6, 4], ar[7, 1], ar[8, 2]]
82 Diag[ar[3, 6], ar[4, 5], ar[7, 1], ar[8, 2]]
83 Diag[ar[3, 6], ar[5, 4], ar[7, 1], ar[8, 2]]
84 Diag[ar[4, 5], ar[6, 3], ar[7, 1], ar[8, 2]]
85 Diag[ar[5, 4], ar[6, 3], ar[7, 1], ar[8, 2]]
86 Diag[ar[1, 8], ar[5, 2], ar[6, 4], ar[7, 3]]
87 Diag[ar[1, 8], ar[2, 6], ar[4, 7], ar[5, 3]]
88 Diag[ar[1, 8], ar[2, 6], ar[5, 3], ar[7, 4]]
89 Diag[ar[1, 8], ar[4, 3], ar[5, 7], ar[6, 2]]
90 Diag[ar[1, 8], ar[4, 3], ar[6, 2], ar[7, 5]]
91 Diag[ar[1, 8], ar[3, 5], ar[4, 7], ar[6, 2]]
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92 Diag[ar[1, 8], ar[3, 5], ar[6, 2], ar[7, 4]]
93 Diag[ar[1, 8], ar[4, 7], ar[5, 3], ar[6, 2]]
94 Diag[ar[1, 8], ar[5, 3], ar[6, 2], ar[7, 4]]
95 Diag[ar[1, 8], ar[3, 7], ar[4, 5], ar[6, 2]]
96 Diag[ar[1, 8], ar[3, 7], ar[5, 4], ar[6, 2]]
97 Diag[ar[1, 8], ar[4, 5], ar[6, 2], ar[7, 3]]
98 Diag[ar[1, 8], ar[5, 4], ar[6, 2], ar[7, 3]]
99 Diag[ar[1, 8], ar[2, 7], ar[3, 5], ar[6, 4]]
100 Diag[ar[1, 8], ar[2, 7], ar[4, 6], ar[5, 3]]
101 Diag[ar[1, 8], ar[2, 7], ar[5, 3], ar[6, 4]]
102 Diag[ar[1, 8], ar[2, 7], ar[3, 6], ar[4, 5]]
103 Diag[ar[1, 8], ar[2, 7], ar[3, 6], ar[5, 4]]
104 Diag[ar[1, 8], ar[2, 7], ar[4, 5], ar[6, 3]]
105 Diag[ar[1, 8], ar[2, 7], ar[5, 4], ar[6, 3]]
106 Diag[ar[1, 8], ar[3, 5], ar[6, 4], ar[7, 2]]
107 Diag[ar[1, 8], ar[4, 6], ar[5, 3], ar[7, 2]]
108 Diag[ar[1, 8], ar[5, 3], ar[6, 4], ar[7, 2]]
109 Diag[ar[1, 8], ar[3, 6], ar[4, 5], ar[7, 2]]
110 Diag[ar[1, 8], ar[3, 6], ar[5, 4], ar[7, 2]]
111 Diag[ar[1, 8], ar[4, 5], ar[6, 3], ar[7, 2]]
112 Diag[ar[1, 8], ar[5, 4], ar[6, 3], ar[7, 2]]
113 Diag[ar[5, 2], ar[6, 4], ar[7, 3], ar[8, 1]]
114 Diag[ar[2, 6], ar[4, 7], ar[5, 3], ar[8, 1]]
115 Diag[ar[2, 6], ar[5, 3], ar[7, 4], ar[8, 1]]
116 Diag[ar[4, 3], ar[5, 7], ar[6, 2], ar[8, 1]]
117 Diag[ar[4, 3], ar[6, 2], ar[7, 5], ar[8, 1]]
118 Diag[ar[3, 5], ar[4, 7], ar[6, 2], ar[8, 1]]
119 Diag[ar[3, 5], ar[6, 2], ar[7, 4], ar[8, 1]]
120 Diag[ar[4, 7], ar[5, 3], ar[6, 2], ar[8, 1]]
121 Diag[ar[5, 3], ar[6, 2], ar[7, 4], ar[8, 1]]
122 Diag[ar[3, 7], ar[4, 5], ar[6, 2], ar[8, 1]]
123 Diag[ar[3, 7], ar[5, 4], ar[6, 2], ar[8, 1]]
124 Diag[ar[4, 5], ar[6, 2], ar[7, 3], ar[8, 1]]
125 Diag[ar[5, 4], ar[6, 2], ar[7, 3], ar[8, 1]]
126 Diag[ar[2, 7], ar[3, 5], ar[6, 4], ar[8, 1]]
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127 Diag[ar[2, 7], ar[4, 6], ar[5, 3], ar[8, 1]]
128 Diag[ar[2, 7], ar[5, 3], ar[6, 4], ar[8, 1]]
129 Diag[ar[2, 7], ar[3, 6], ar[4, 5], ar[8, 1]]
130 Diag[ar[2, 7], ar[3, 6], ar[5, 4], ar[8, 1]]
131 Diag[ar[2, 7], ar[4, 5], ar[6, 3], ar[8, 1]]
132 Diag[ar[2, 7], ar[5, 4], ar[6, 3], ar[8, 1]]
133 Diag[ar[3, 5], ar[6, 4], ar[7, 2], ar[8, 1]]
134 Diag[ar[4, 6], ar[5, 3], ar[7, 2], ar[8, 1]]
135 Diag[ar[5, 3], ar[6, 4], ar[7, 2], ar[8, 1]]
136 Diag[ar[3, 6], ar[4, 5], ar[7, 2], ar[8, 1]]
137 Diag[ar[3, 6], ar[5, 4], ar[7, 2], ar[8, 1]]
138 Diag[ar[4, 5], ar[6, 3], ar[7, 2], ar[8, 1]]
139 Diag[ar[5, 4], ar[6, 3], ar[7, 2], ar[8, 1]]
```

Finding EWs...

Computing mat...

```
{139, 69 114}
```

Computing Rank...